Property Determinations of Open Clusters using Hierarchical Bayesian Modelling and Neural Networks

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**Abstract:**

**Context:** Open clusters are very useful for age and metallicity estimations, which makes that excellent for tracing the evolution of galaxies. Current methods of estimating open cluster properties are time consuming and heavily rely on literature values. We put forward that a more efficient method is by using a framework consisting a neural network inside an HBM.   
**Aims:** To determine accurate properties of open clusters, using a neural network to convert fundamentals and observables, which can be used by a hierarchical Bayesian model to simultaneously constrain the fundamental variables and give population wide mean values of these properties.   
**Methods:** We trained a deep feedforward neural network to convert fundamentals: age, mass, [Fe/H], initial helium and mixing length to observables: effective temperature, stellar radius and Δν using a grid of stellar evolution tracks calculated using MESA (B. Paxton et al. (2010) [MESA]). This was then incorporated into an HBM and property estimations were made using this framework given suitable priors on the fundamentals and observational data for different clusters, taken from the literature.   
Results:  
Conclusion:

Estimated Word Count:   
Key words: neural network, HBM, Bayesian, M67

Contents:

Nomenclature:

**Introduction:**  
Open clusters are one of the most important types of objects when it comes tracing the evolutionary history of our galaxy, which is still very much an open topic of research as making accurate property determinations can be quite challenging but is crucial to figuring out the mechanisms behind galactic evolution. Open clusters are loosely gravitationally bound groups of typically a few hundred coeval stars, meaning all the stars formed at approximately the same time in a relatively small region of a galaxy where there was a particular chemical composition. The result of this being that open cluster members can be treated as having the same age and [Fe/H] (hereafter metallicity). This feature is hugely beneficial for property determinations over single star measurements as by measuring open cluster members the mean age and metallicity of these effectively converges to the true values (see \cite{Hippel\_2005} and \cite{salaris\_2004} for more details). The most common method of doing this is by isochrone fitting, where an isochrone is a plot on a Herzsprung-Russell which describes stars of the same age, and typically also keeps the chemical composition constant. Isochrone fitting requires sampling ages and metallicities and converting those fundamentals parameters to observables (effective temperature, luminosity etc.) such that they can be compared to the observed data for the open cluster. Depending on the method the isochrones are evaluated to determine how well each isochrone fits and the best isochrone’s age and metallicity are taken to be true for the cluster. However, certain choices for the method of sampling and parameter conversion tool, can cause the fitting process to be incredibly time consuming. There are a variety of sampling methods from fitting by-eye \citep{brandt \_2015} to automated fitting processes using software \citep(perren\_2015) and Bayesian analysis (\cite{hills\_2015}, \cite{jeffery\_2016}, \cite{jørgensen\_2005}). Which compounds upon the time for the conversion process from fundamentals to observables which in most studies to date has been done using stellar evolution models to generate the isochrones via frameworks like MESA (\cite{ball\_2017}, \cite{ball\_2018}, \cite{MESA}) and DSED (\cite{jeffery\_2016}, \cite{DSED}). These require generating a stellar evolutionary track for each sampled age and metallicity which can take hours per track generated, although some studies (see \cite{jørgensen\_2005}) skirt this by using a pre-existing set of tracks and interpolating between them but raises different problems. There are other open cluster dating methods like detached eclipsing binaries (\cite{brewer\_2016} and \cite{bavarsad\_2016}) and white dwarf cooling (\cite{kalirai\_2001}, \cite{bedin\_2015}) but these outside the scope of this study.

The purpose of this study is to improve upon the isochrone fitting methods stated above by avoiding lengthy parameter conversion and track interpolation by training a neural network on MESA tracks to convert fundamental to observable parameters (SEE NEURAL NETWORK SECTION FOR MORE DETAILS). The sampling is to be done using a hierarchical Bayesian model (HBM) where literature values inform the sampling such that minimal time is wasted sampling in parameter space unlikely to yield the true values. This method should allow for sampling of open clusters to be done much faster with the added benefit that the HBM allows for simultaneous constraint of a large number of variables and is able to estimate the spread of those variables, which haven’t been measured before for open clusters (SEE THE HBM SECTION FOR FURTHER DETAILS).

**Target selection and data collection:**  
We selected 6 open clusters (NGC 2682 also known as M67, NGC 6791, NGC 6819, Ruprecht 147, NGC 752, NGC 188) which have a large range of ages and metallicities that have been relatively well studied, see Table 1 for details on the results of previous studies. To select stars to study from these open clusters we began by finding membership studies (calculations of the probability that a star belongs to a particular open cluster), from which we chose the membership study of M67 done by \cite{gao\_2018} and for the other 5 clusters we used the work of \cite{cantat-gaudin\_2018}.

What follows are the steps and measures used to discard stars and gather data to calculate luminosities.   
- These 2 membership papers contained GAIA DR2 IDs, which allowed us to query GAIA DR2 \citep{GAIA\_DR2} to get GAIA G-band apparent magnitudes and distance estimates for the stars calculated by \cite{bailer-jones\_2018}. We then discarded stars without G-band measurements.  
- Effective temperatures, Teff, are crucial for this study so we wanted to get accurate and consistent Teff measurements whenever possible. Some of the most accurate Teff determinations have been made using the infrared spectroscopy of the Apache Point Observatory Galactic Evolution Experiment (APOGEE) \citep{APOGEE}, which we queried through SDSS DR12. Unfortunately, APOGEE didn’t have effective temperatures for all the stars we were considering, so for the remaining stars we queried the SIMBAD database and discarded stars without a Teff from APOGEE or SIMBAD.   
- We then selected the greatest number of stars from the OCs that gave a mean membership uncertainty of 1%.   
- For stars that didn’t have APOGEE Teffs but multiple literature Teffs, we counted anomalies as stars whose Teffs differed from the median literature value by greater than 90k, which were removed. The Teff of these stars were then selected by the paper which provided the largest number of Teffs for that OC of the papers that provided Teffs for each star.   
- The absolute GAIA G-band magnitude was calculated as follows:

$ M\_G = m\_G + 5 - \log\_{10}(r) - A\_G $ eq.1

where r is the median of the distance estimates for the cluster and $A\_G$ is the line-of-sight extinction ignoring the effects of reddening, calculated by:

$ A\_G = R(G)E’ $ eq.2

Where R(G) is the GAIA G-band extinction coefficient (2.294) determined by \cite{sanders\_2018} and E’ is the dust reddening taken from the best fit values in the bayestar2019 dust map. The GAIA G-band bolometric correction, $BC\_G$, was taken from a model dependent only Teff, by \cite{andrae\_2018}. From which we could get the bolometric magnitude:

$ M\_Bol = BC\_G + M\_G $ eq.3

And then we find the luminosity (in units of solar luminosities) by:

$ L = 10^{-0.4(M\_{Bol} - M\_{Bol, \odot})} $ eq.4

Where $M\_{Bol, \odot}$ is taken to be 4.75 (\cite{jordi\_2010}, \cite{casagrande\_2018}). We then plotted the HR-diagram and removed blue stragglers, binaries and other stars we thought might interfere with the HBM sampling. We removed the blue stragglers because they cannot be fitted to an isochrone as their properties evolve in an atypical way. Binaries are removed because their luminosities shift them far away from the line an isochrone would be fitted to and are removed like the other anomalies to assist with HBM sampling.

**Table 1**: Literature search on the open clusters in this study. HQS – High Quality Spectroscopy, LQS – Low Quality Spectroscopy, Phot – Photometric

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| cluster | Age estimation, Gyr | Metallicity, [Fe/H] |
| NGC 2682 | 3.64 \citep{Bossini}  3.45 ± 1.13 \citep{Netopil \_2016} | 0.03 (HQS) \citep{Netopil \_2016} 0.00 (LQS) \citep{Netopil \_2016} -0.01 (Phot) \citep{Netopil \_2016} |
| NGC 6791 | 8.45 \citep{Bossini}  7.00 ± 2.46 \citep{Netopil \_2016} | +0.42 (HQS) \citep{Netopil \_2016} +0.35 (LQS) \citep{Netopil \_2016} +0.44 (Phot) \citep{Netopil \_2016} |
| NGC 6819 | 2.00 \citep{Bossini}  2.11 ± 0.44 \citep{Netopil \_2016}  2.5 \citep{balona\_2013}  2.25 ± 0.20 \citep{bedin\_2015} | +0.09 (HQS) \citep{Netopil \_2016}  -0.04 (LQS) \citep{Netopil \_2016}  +0.03 (Phot) \citep{Netopil \_2016} |
| Ruprecht 147 | 2.5 ± 0.25 \citep{curtis\_2013}  3 \citep{curtis\_2013}  3.25 \citep{curtis\_2013} | +0.08 \citep{bragaglia\_2018} |
| NGC 752 | 1.48 \citep{Bossini}  1.69 ± 0.66 \citep{Netopil \_2016} | -0.03 (HQS) \citep{Netopil \_2016}  -0.09 (LQS) \citep{Netopil \_2016}  -0.04 (Phot) \citep{Netopil \_2016} |
| NGC 188 | 7.53 \citep{Bossini}  6.27 ± 2.30 \citep{Netopil \_2016}  5.78 ± 0.03 \citep{hills\_2015}  6.45 ± 0.04 \citep{hills\_2015} | +0.11 (HQS) \citep{Netopil \_2016}  -0.02 (LQS) \citep{Netopil \_2016}  -0.02 (Phot) \citep{Netopil \_2016}  +0.125 ± 0.003 \citep{hills\_2015}  −0.077 ± 0.003 \citep{hills\_2015} |

**MESA stellar model**  
To generate a grid of stellar tracks to train a neural network on we chose to use Modules for Experiments in Stellar Astrophysics (MESA), which uses a specific model of stellar physics in order to evolve stars from some initial conditions and give their properties at a range of points across their lifetime (see \cite{MESA}, \cite{MESA2} and \cite{MESA3}). As our understanding of stellar evolution isn’t perfect, our stellar models aren’t perfect and as such there are simplifications must be made. An example is that the grid used in this study doesn’t account for stellar rotation which has knock on effects on temperature, luminosity and asteroseismic values (among others). However, studies like this assists in refining models by testing them.

**Table 2**: MESA tracks initial input parameter ranges

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| --- | --- | --- |
| Input Parameter | range | increments |
| M / M\_{\odot} | 0.8 - 1.68 | 0.04 |
| [Fe/H]\_0 | -0.8 - 0.6 | 0.2 |
| Y\_0 | 0.24 - 0.32 | 0.02 |
| MLT | 1.7 - 2.3 | 0.2 |

A more detailed description of the construction of the MESA grid can be found in \cite{li\_2018}, the only difference being in the initial input parameter ranges, shown here in Table 2.

**Working with Neural Networks:  
 Machine Learning and Neural Networks:**  
Machine learning is the term used to describe a broad spectrum of statistical tools used to identify or reproduce different kinds of trends. There are many different types of machine learning methods because not all methods are suitable for the limitless number of machine learning applications. For instance, in this study we will focussed on supervised machine learning where we present a trend that we want the neural network to be able to reproduce. This is in the form of the MESA grid where we have a set of input parameters we know map onto the output parameters (which can be seen in some figure out I guess I must eventually put in an HR diagram/isochrone)

Initial conditions in a grid of:  
o Age = 1 to 12 Gyr

3859 tracks in total  
2659783 datapoints

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| Ted von Hippel: <https://arxiv.org/pdf/astro-ph/0509152.pdf> |
| M. Salaris et al. 2004: “The age of the oldest Open Clusters”  **Cluster age estimates in table 1**  <https://www-aanda-org.ezproxye.bham.ac.uk/articles/aa/full/2004/04/aah4736/aah4736.html>  The most reliable stellar ages are obtained for the star clusters belonging to the various populations, i.e., the globular clusters (GCs) in the halo, thick disk and bulge, and the open clusters (OCs) in the thin disk. The advantage of dating star clusters over individual stars – whose age determination relies entirely on the knowledge of individual metallicities, effective temperatures and gravities (or absolute magnitudes), which have to be fitted by the appropriate theoretical model – stems from the fact that star clusters are made of coeval objects, largely with the same initial chemical composition and located at the same distance, so that it is possible to use morphological parameters deduced from theoretical isochrones in order to derive their age. |
| Clio Bertelli Motta et al. 2016: “Observing the products of stellar evolution in the old open cluster M67 with APOGEE”  **Isochrone fitting M67**  <https://academic-oup-com.ezproxye.bham.ac.uk/mnras/article/466/2/2161/2687805> |
| D. Stello, et al. 2016: “THE K2 M67 STUDY: REVISITING OLD FRIENDS WITH K2 REVEALS OSCILLATING RED GIANTS IN THE OPEN CLUSTER M67”  **Isochrone fitting/grid based modelling**  <https://openresearch-repository.anu.edu.au/bitstream/1885/152166/2/01_Stello_THE_K2_M67_STUDY%253A_REVISITING_2016.pdf> |
| 1. F. Oliveira, et al. 2013: “Fitting isochrones to open cluster photometric data III. Estimating metallicities from UBV photometry”   **Isochrone fitting metallicity using the cross-entropy global optimization algorithm contains explanation**  “The metallicity is a critical parameter that affects the correct determination of fundamental characteristics of a stellar cluster and has important implications in Galactic and stellar evolution research. Fewer than 10% of the 2174 currently atalogued open clusters have their metallicity determined in the literature. In this work we present a method for estimating the metallicity of open clusters via non-subjective isochrone fitting using the cross-entropy global optimization algorithm applied to UBV photometric data.”  “Important questions that depend on metallicity, which is usually measured by the [Fe/H] ratio, are the determination of chemical abundance gradients (see Lépine et al. 2011, and references therein), determination of the rotational speed of the spiral pattern, and the co-rotation radius (Dias & Lépine 2005), and in the stellar context the empirical determination of the initial mass function, among many other fields of study.”  “Very schematically, the CE procedure provides a simple adaptive way of estimating the best-fit parameters. It involves an iterative procedure that follows the steps outlined below:  - random generation of the initial sample of fit parameters, respecting predefined criteria;  - selection of the best candidates based on calculated weighted likelihood values;  - generation of a random fit parameter sample derived from a new distribution based on the previous step;  - repeat until convergence or stopping criteria reached.” |
| Monteiro, H.; Dias, W. S.; Caetano, T. C. 2010: “Fitting isochrones to open cluster photometric data. A new global optimization tool”  **Isochrone fitting using cross-entropy global optimization algorithm ORIGINAL**  <https://ui.adsabs.harvard.edu/abs/2010A%26A...516A...2M/abstract> |
| H. Pöhnl and E. Paunzen 2010: “A statistical method to determine open cluster metallicities”  Method which differs from isochrone fitting as it doesn’t really on metallicity measurements.  “The study of open cluster metallicities helps to understand the local stellar formation and evolution throughout the Milky Way. Its metallicity gradient is an important tracer for the Galactic formation in a global sense. Because open clusters can be treated in a statistical way, the error of the cluster mean is minimized.”  <https://www-aanda-org.ezproxye.bham.ac.uk/articles/aa/pdf/2010/06/aa10855-08.pdf> |
| L. A. Balona, et al. (2013)  **Isochrone fitting**  “we determine the distance and age of NGC 6819 using several different methods. From isochrone fitting we find the age of the cluster to be about 2.5 Gyr” |
| L. N. Brewer, et al. (2016): “Determining the Age of the Kepler Open Cluster NGC 6819 With a New Triple System and Other Eclipsing Binary Stars”  **Ageing using Detached Eclipsing Binaries**  “using all measured eclipsing binary star masses and radii, we constrain the  age to 2.38 ± 0.05 ± 0.22 Gyr. The quoted uncertainties are estimates of measurement and systematic uncertainties (due to model physics differences and metal content), respectively”  “Measurements of the masses and radii of the component stars in detached eclipsing binaries (DEB) can be used to precisely determine the age of the stars if at least one of the eclipsing stars has begun to evolve away from the main sequence. The use of mass and radius (M – R)  measurements of eclipsing stars avoids or minimizes systematic uncertainties introduced by factors such as distance, interstellar reddening, and color-temperature conversions that can affect age measurements (Andersen 1991; Torres et al. 2010). When eclipsing binaries occur in star clusters, their utility increases dramatically because they place constraints on the age of all the stars in the cluster. Multiple DEBs in a cluster can provide M – R measurements for stars having a range in mass, and can constrain the age even more tightly”  “Kalirai et al. (2001) describes how the age of NGC 6819 can be found using the faint end of the white dwarf cooling sequence, and Bedin et al. (2015) present an age of 2.25 ± 0.20 Gyr using that technique.”  <https://arxiv.org/pdf/1601.04069.pdf> |
| Bavarsad E. A. et al. (2016) : “THE DETACHED ECLIPSING BINARY KV 29 AND THE AGE OF THE OPEN CLUSTER M11”  **More detailed description of how age determinations work with DEBs**  <https://iopscience.iop.org/article/10.3847/0004-637X/831/1/48> |
| Kalirai JS, et al. (2001)  “The CFHT Open Star Cluster Survey. II. Deep CCD photometry of the old open star cluster NGC 6819”  **Age determinations using white dwarf cooling** |
| Bedin L. R. et al. (2015)  “Hubble Space Telescope observations of the Kepler-field cluster NGC 6819 – I. The bottom of the white dwarf cooling sequence”  **Age determination of NGC 6819 via white dwarf cooling**  “2.25 ± 0.20 Gyr”  <https://academic-oup-com.ezproxye.bham.ac.uk/mnras/article/448/2/1779/1056101> |
| Lund M. N. et al. (2016): “Asteroseismology of the Hyades with K2: first detection of main-sequence solar-like oscillations in an open cluster”  **first ever detections of solar-like oscillations in main-sequence stars in an open cluster**  **used kepler** |
| Hekker, S. et al. (2011): “Asteroseismic inferences on red giants in open clusters NGC 6791, NGC 6819, and NGC 6811 using Kepler”  **Asteroseismology of OC red giants using Kepler** |
| EMBEDDED CLUSTERS IN MOLECULAR CLOUDS  <https://pdfs>.semanticscholar.org/dd06/a8a9143f9e168dc2dc30c8cc3a15cc10fb40.pdf  galactic clusters form in giant molecular clouds (GMCs) and during their formation and earliest stages of evolution are completely embedded in molecular gas and dust, and thus obscured from view. Given the constraints imposed by traditional techniques of optical astronomy, direct observation and study of young embedded clusters had been extremely difficult, if not impossible. However, during the past two decades the development of infrared astronomy and, more recently, infrared array detectors, has dramatically improved this situation. |
| Jeffery, E. J. et al. (2016): “A BAYESIAN ANALYSIS OF THE AGES OF FOUR OPEN CLUSTERS”  **Isochrone fitting using Bayesian MCMC and Dartmouth Stellar Evolution Database (Dotter**  **et al. 2008) to create tracks.**  “Star clusters have long been important tools for studying  stellar evolution, specifically because they play the pivotal role  in determining the ages of stars. The most commonly used  method for measuring the age of an open star cluster involves  fitting an isochrone to the cluster’s observed color–magnitude  diagram (CMD), specifically to the cluster’s main sequence  turn-off (MSTO). Generating and fitting isochrones to a cluster  CMD to determine its age also requires knowledge of the  cluster’s metallicity, distance, and reddening. Oftentimes,  finding a best fit of these three parameters (plus age) is a  subjective process, as some of these parameters are correlated  with each other. This difficulty is reflected in isochrones that  appear to fit the CMD equally well with various combinations  of cluster parameters (see, for example, Figure 2 of VandenBerg & Stetson 2004). Moreover, the fit of the MSTO can be  challenging and isochrones may give inconsistent results in  different filters, even when using the same cluster parameters  (see, for example, Figure 10 of Sarajedini et al. 1999).”  <https://iopscience.iop.org/article/10.3847/0004-637X/828/2/79/pdf> |
| Hills, S. et al. (2015)  **Bayesian isochrone fit of NGC 188**  <https://iopscience.iop.org/article/10.1088/0004-6256/149/3/94/pdf> |
| Brandt, T. D. et al. (2015): “Rotating Stellar Models Can Account for the Extended Main Sequence Turnoffs in Intermediate Age Clusters”  **Isochrone fitting “by-eye”**  <https://arxiv.org/abs/1504.04375> |
| Perren, G. I. et al. (2015)  Automated isochrone fitting  <https://arxiv.org/pdf/1412.2366.pdf> |
| Appourchaux et al. (2015) |
| Ball and Gizon, (2017)  MESA  Ball, W. H. et al. (2018)  MESA |
| jørgensen\_2005  Bayesian age determinations |
| cantat-gaudin\_2018  **membership paper with 1k clusters**  <https://www.aanda.org/articles/aa/full_html/2018/10/aa33476-18/aa33476-18.html> |
| gao\_2018  M67 membership found using random forest  <https://ui.adsabs.harvard.edu/abs/2018ApJ...869....9G/abstract> |
| sanders\_2018  **where we get the G-band extinction coefficient**  <https://arxiv.org/pdf/1806.02324.pdf> |
| green\_2019  **3D dustmap**  <https://iopscience.iop.org/article/10.3847/1538-4357/ab5362> |
| andrae\_2018  **Teff 🡪 bolometric correction model** |
| jordi\_2010 and casagrande\_2018  **sources for solar bolometric magnitude** |
| li\_2018  Tanda’s training grid setup using MESA  <https://arxiv.org/pdf/1810.13015.pdf> |
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CONCERNS:  
- introduction doesn’t discuss papers but more references methods that those papers use  
- make sure to properly subscript Teff and A\_G etc.  
- in acknowledgements do we have to acknowledge keras and MESA, GAIA, pymc3 etc.?  
- **for project work 2 I suggest you make the Github repo public and submit the link to the repo.**

- limitations of using single G-band extinction coefficient